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A Study on Differences of High School Students' Mathematical Modeling Cognition

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Applying verbal protocol analysis and expert-novice research methods, this study explores the distinguishing features of cognition in mathematical modeling between 15 excellent students and 15 normal students. The results showed some different cognitive features in problem representation, strategy applied, thought, result and efficiency of mathematical modeling of higher students at different levels.

Key words: mathematical modeling, cognitive features, method of verbal protocol analysis, expert-novice research

Due to the powerful educational value of mathematical modeling, many countries have put this content into their mathematics curriculum (Werner, Galbraith, & Mogen, 2006). For that reason, mathematical modeling has been introduced into the high school mathematics curriculum according to *the Standard for Senior High School Mathematics Curriculum (experimental version)* enacted in 2003. Although some progress has been made in the execution of this curriculum, educators are not satisfied with the curriculum implementation effects (Li & Yu, 2008). One of the major causes is that it's hard to provide necessary cognitive psychological instructions for the designing and teaching of this curriculum, which results from the absence of research on high school students' mathematical modeling cognition patterns. In recent years, much research has been performed on university students' mathematical modeling cognition, and principles and strategies have also been put forward (Li, 2007; Li, Pang, & Yu, 2009; Li, Yu, & Song, 2008; Li, Yu, & Zhang, 2008; Li, Yu, & Pang, 2009), which provide effective teaching with influential factors. Based on the experience and methods mentioned above, some elementary progress has been made in the research on high school students' mathematical modeling cognition (Li, 2009; Li, Cai, & Wang, 2010).

In this study, we compared mathematical the modeling cognitive features of high school students at different levels with verbal protocol analysis and expert-novice research, aiming at revealing cognitive features and differences of students, and providing some psychological basis for the designing and teaching of mathematical modeling and improving its teaching effects.

Methodology

Participants

The participants in the current study are 218 students from four high schools (two high-level schools and two normal schools) from Jiangsu and Guangdong province. In each school one class was selected in the 12th grade. *Mathematical Modeling Testing* was performed (Li, 2009). Students scoring in the top 15 were selected as excellent students (these subjects which took more mathematical modeling training are treated as expert subjects in our research) while those from 73%-27% are chosen randomly as normal students (these subjects which took less mathematical modeling training are treated as novice subjects in our research).

Instruments

In light the features and requirements of verbal protocol test time and mode, six problems about mathematical modeling were, among the eight original problems, which have been screened, adjusted and improved by experts and teachers (Li, 2009). The first two problems are for practice and the last four are for testing. Among the four problems, the third one is simple, the forth is moderate and the fifth and sixth are complicated. For the sixth problem, subjects are not required to complete the solution, but to explain their ideas; the results are not counted but are used as reference materials for the thinking feature analysis. The instruction for the test is, “Please read the problems aloud and say whatever you think when solving the problems. You needn’t say why you do it like this, but you have to say what is your thinking progress. I will be recording you in order to figure out how you deal with the problems.”

Data Collection and Data Analysis

Verbal protocol practice. The tester explained the instruction requirements to the subjects in advance and made a demonstration using the first problem; then the subject was required to practice with the second question. If a long pause occurred, the tester could give him (her) a prompt : “What are you thinking?” “Please speak aloud what you are thinking.” to spur him (her) to voice his (her) thinking and give a timely, continuous and complete protocol.

Verbal protocol tests. The formal tests started after the subject's observation, practice and adaptation to mathematical modeling protocol test and the time was unlimited. After the subject finished solving the problems(or gave up solving), the tester recorded the time spent, required the subject to recall and describe their ideas and process timely, then the tester asked interviewed the subject and asked questions about any unclear comments.

Sorting of verbal protocol date. Each subject's verbal protocol recording was organized into a preliminary text when the tests ended. Then the tester checked and complemented the preliminary text along with the recording, paper-and-pencil test and interview materials. As a result, complete mathematical modeling verbal protocol materials were formed in the text mode.

Comparing and analyzing verbal protocol materials. Through expert-novice comparison, mathematical modeling verbal protocol materials and interview materials of the excellent and the normal participates or students were analyzed from the aspects of representation, strategy adoption, thinking patterns, results and solving efficiency. Cognitive features of the two groups were compared on the basis of classified statistic of the excellent and normal students.

Results

Comparison of Representation Features

Based on subjects' verbal protocol materials, comparison of mathematical modeling representation mode, span and method has been made between excellent students and normal ones. Results are shown in Table 1.

Table 1

The comparison of representation features of mathematical modeling problems between excellent students and normal students

Groups No.	Problem		Mode			Span		Method
	SR	MR	MeR		SiR	PlR	OR	CR
EG	45	45	45	32	8	37	10	35
NG	45	45	45	11	34	11	36	9
Z	—	—	4.316		−4.887	4.863	−4.871	4.879
p			0.005		0.001	0.001	0.001	0.001

Note: EG, Excellent Group; NG, Normal group; SR, Symbolic Representation; MR, Methodical Representation; MeR, Mechanical Representation; SiR, Single Representation; PlR, Plural Representation; OR, One-way Representation; CR, Circular Representation.

According to the analysis of subjects' verbal protocol materials, we think that Symbolic Representation, Methodical Representation and Mechanical Representation are included in representation modes of subjects' mathematical modeling problems. Symbolic representation means comprehending and solving mathematical modeling problems from words, images, and mathematical symbols, this representation is about acquisition and comprehension of descriptive knowledge. Methodical representation means comprehending and solving mathematical modeling problems from strategies and methods; it's about acquisition and comprehension of programmed knowledge. Mechanical representation means comprehending and solving mathematical modeling problems from structures contained and theories needed; it's about knowledge and structures activation of theories and categories on mathematical modeling problems. As shown in table 1, there is significant difference in representation modes of mathematical modeling problems between excellent students and normal ones. Although both groups have used Symbolic Representation and Methodical Representation, excellent students tend to use more Mechanical Representation than normal students when solving problems.

According to the analysis of subjects' verbal protocol materials, we consider that the representation span of subjects' mathematical modeling problems can be depicted by Plural Representation and Single Representation. Plural Representation is that a modeler uses Plural Representation modes (such as reading, diagrammatizing, and symbolization) to apperceive and comprehend information, connotation and structures of mathematical modeling problems. Single Representation means that a modeler only uses one of the representation modes. As shown in Table 1, significant difference does exist in the representation span of mathematical modeling problems between excellent students and normal students. Excellent students are more likely to use Plural Representation while normal ones are inclined to use Single Representation.

According to the analysis of subjects' verbal protocol materials, we consider that Circular Representation methods and One-way Representation are included in the representation methods. Circular representation refers to subjects using representation many times and repeatedly during the process of mathematical modeling. One-way representation refers to subjects not repeating any more after using representation in the stage of problem comprehending. Table 1 showed that significant difference in representation methods of mathematical modeling problems occurs between excellent students and normal students. Excellent students tend to use Circular Representation methods while normal ones are inclined to use One-way Representation methods.

The Comparison of Strategic Features

Based on subjects' protocol test materials, analysis and statistic comparison of strategic types used in mathematical modeling, comparisons have been made between excellent students and normal students. Results are shown in table 2.

From Table 2, we can conclude that excellent students and normal students differ greatly in the application of Mathematical Modeling Assumption, Mathematical Modeling Construction, Mathematical Modeling Self-monitoring, Mathematical Modeling Validation and Mathematical Modeling Adjustment Strategy.

Mathematical Modeling Assumption Strategy is used to assume the perspicuity and idealization of the situations, conditions and objectives of the real problems. Through the analysis, the Mathematical Modeling Assumption Strategy used by subjects can be categorized into Feasible Assumption Strategy, Accurate Assumption Strategy and Equilibrium Assumption Strategy. Feasible Assumption Strategy refers to an approximate and simplified assumption strategy that the modeler uses in order to pursue the feasibility of the mathematical modeling problems. The adoption of this strategy tends to make simple assumptions that some inaccuracies must occur between the real problems and the model. Accurate Assumption Strategy refers to an accurate and practical assumption strategy that the modeler uses in order to achieve the accuracy of the mathematical modeling problems. The adoption of this strategy tends to make complicated assumptions that the process and results of the modeling may be too complex or even too difficult to achieve. Equilibrium Assumption Strategy refers to a moderate and balanced assumption strategy that the modeler uses after considering both feasibility and accuracy of the mathematical modeling problems. The adoption of this strategy is beneficial not only to the smooth development of the mathematical modeling, but also for better solutions to real problems. Table 2 indicates that excellent students tend to choose the Equilibrium Assumption Strategy, while normal students choose the Accurate Assumption Strategy. However, there are little differences in their adoption of the Feasible Assumption Strategy.

Mathematical Modeling Construction Strategy is the strategy adopted for practical problems modeling. Through the analysis of the subjects' protocol materials, Mathematical Modeling Construction Strategy used by subjects can be categorized into Pattern Recognition Match Strategy, Sample Analog Strategy and Instant Generation Strategy. Pattern Recognition Match Strategy refers to a construction strategy in which the student matches existing problems mentally with current problems in order to recognize the mode of current problems and put the model or methods directly into use.

Table 2

The Comparison of Mathematical Modeling Problems Strategic Features between Excellent Students and Normal Students

Groups	MMAS			MMCS		
	FS	AS	ES	PRS	SAS	IGS
EG	9	6	30	9	24	12
NG	6	27	12	6	6	33
χ^2	11.817			11.431		
P	0.001			0.001		

Groups	MMSMS			MMVS		
	IMS	SEMS	RMS	DVS	TDS	IJS
EG	27	6	12	8	18	19
NG	3	12	30	28	9	8
χ^2	18.428			14.465		
P	0.001			0.001		

Groups	MMAJS		
	AAS	MMEAS	MSAS
EG	18	18	9
NG	9	9	27
χ^2	13.859		
P	0.001		

Note: EG, Excellent Group; NG, Normal Group; MMAS, Mathematical Modeling Assumption Strategy; MMCS, Mathematics Model Construction Strategy; FS Feasible Strategy; AS, Accurate Strategy; ES, Equilibrium Strategy; PRS, Pattern Recognition Match Strategy; SAS, Sample Analog Strategy; IGS, Instant Generation Strategy; MMSES, Mathematical Modeling Self-monitoring Strategy; MMVS, Mathematical Modeling Validation Strategy; IMS, Instant Monitoring Strategy; SEMS, Selective Monitoring Strategy; RMS, Review Monitoring Strategy; DVS, Data Validation Strategy; TDS, Theoretical Deduction Strategy; IJS, Intuitive Judgment Strategy; MMAJS Mathematical Modeling Adjustment Strategy; AAS, Assumptive Adjustment Strategy; MMEAS, Modeling Methodical Adjustment Strategy; MSAS, Model Solving Adjustment Strategy.

Sample Analog Strategy refers to a construction strategy of drawing an analogy between current problems and samples with similar patterns, modeling methods and existing models. That is, in the process of mathematical modeling, the modeler recalls the problems which he or she once solved successfully and which are analogous with current problems, analyzes the similarities and differences between previous problems and current ones, and applies the previous thoughts, methods and results to current problems with improvements. Instant Generation Strategy refers to a strategy of exploring mathematical modeling with free thinking about current problems. Table 2 indicates that excellent students are inclined to use Sample Analog Strategy, while normal students prefer Instant Generation Strategy. However, little difference exists in their adoption of the Pattern Recognition Match Strategy.

Mathematical Modeling Self-monitoring Strategy refers to a strategy in which the student plans examines, evaluates, uses feedback, adjusts and monitors his (her) cognitive activity in the process of mathematical modeling. Through the analysis of subjects' protocol materials, we can categorize the Mathematical Modeling Self-monitoring Strategy into Instant Monitoring Strategy, Selective Monitoring Strategy and Review Monitoring Strategy. Instant Monitoring Strategy means that the student continuously monitors and controls every step and result when modeling. Selective Monitoring Strategy means that the student selectively monitors and controls some steps and result when modeling. Review Monitoring Strategy means that the student monitors and controls all steps and results after finishing preliminary modeling. Table 2 indicates that excellent students tend to use Instant Monitoring Strategy, while normal students use Selective Monitoring Strategy and Review Monitoring Strategy.

Mathematical Modeling Validation Strategy refers to the student's validation of the constructed models and the correctness and rationality of the results. Through the analysis, we can categorize Mathematical Modeling Validation Strategy into Data Validation Strategy, Theoretical Deduction Strategy and Intuitive Judgment Strategy. Data Validation Strategy means validating the constructed models and the correctness and rationality of the results by checking the data involved or related to the problem. Theoretical Reduction Strategy refers to the strategy of validating the constructed models and the correctness and rationality of the results by checking the existing theoretical reductive steps of the mathematical modeling. Intuitive Judgment Strategy means validating the constructed models and the correctness and rationality of the results through the student's intuition and perception. Table 2 indicates that excellent students tend to use Theoretical Deduction Strategy and Intuitive Judgment Strategy, while normal students use Data Validation Strategy.

Mathematical Modeling Adjustment Strategy refers to a strategy in which the student adjusts some steps of the mathematical modeling when facing some obstacles or mistakes. Through the analysis of the subjects' protocol materials, we can categorize Mathematical Modeling Adjustment Strategy into Assumptive Adjustment Strategy, Modeling Methodical Adjustment Strategy and Model Solving Adjustment Strategy. Assumptive

Adjustment Strategy refers to a strategy of adjusting the previous mathematical modeling assumption when facing obstacles in validation. Modeling Methodical Adjustment Strategy refers to a strategy of adjusting the mathematical modeling methods when facing obstacles in validation. Model Solving Adjustment Strategy refers to adjusting solving methods and process of constructed models when the student realizes some deviation exists between practical problems and mathematical modeling results. Table 2 indicates that excellent students tend to use Assumptive Adjustment Strategy and Modeling Methodical Adjustment Strategy, while normal students use Model Solving Adjustment Strategy.

The Comparison of Thinking Patterns

According to the subjects' protocol materials, analyses and statistical comparisons have been made of mathematical modeling thoughts, results and the time consumed between excellent students and normal students. These are shown in Table 3.

Table 3

The Comparison of Thinking Patterns, Results and Efficiency of Mathematical Modeling between Excellent Students and Normal Students

Groups	Problem No.	Thinking patterns			Results		
		TPC	TPS	TPE	T	F	A
EG	45	28	6	7	27	12	6
NG	45	8	30	35	9	31	5
Z		3.667	-4.938	-5.282			
χ^2					11.920		
p		0.001	0.001	0.001		0.001	

Groups	time-consuming of mathematical modeling problem (Min)		
	Easy	Medium	Complex
EG	11.8	18.4	27.5
NG	16.6	26.7	16.3
Z	-3.411	-4.271	4.852
p	0.001	0.001	0.001

Notes: EG, Excellent Group; NG, Normal Group; TPC, Thinking Patterns Conversion ; TPS, Thinking Patterns Set; TPE, Thinking Patterns Errors ; T, True ; F, False; A , abandon .

In Table 3, we can see excellent students and normal students differ a lot in mathematical thoughts, results and efficiency. Excellent students are more likely to converse their thinking patterns, which means excellent students have more flexible mathematical thoughts. The significant difference also exists in their thinking set. Normal students tend to use more thinking sets which indicates that they are more greatly influenced by thinking set than are excellent students when modeling. The difference is also obvious in results achieved by excellent and normal students. The number of reasonable results gained by excellent students is more than those gained by normal students. Significant difference in efficiency also appeared between excellent students and normal students, the time consumed by the excellent student for simple and moderate problems is much shorter than the normal student who solves the same number of problems. It indicates that excellent students have much higher efficiency than normal students.

Discussion

The Differences in Representation Modes

Research has found that there is difference in representation modes of mathematical modeling problems between excellent and normal students. Although both excellent and normal students adopt Symbolic Representation and Methodical Representation, excellent ones use Mechanical Representation more. Moreover, excellent students are able to use Symbolic Representation, Methodical Representation and Mechanical Representation flexibly. First, they use Symbolic Representation to perceive and understand the basic information of the problems; then, they use Mechanical Representation to grasp the key points of the problems; finally, they use Methodical Representation immediately to search for and select ways of thinking and solving the mathematical modeling. On the contrary, normal students usually adopt Methodical Representation immediately after using Symbolic Representation, but they use adopt Mechanical Representation. Even though they have adopted Mechanical Representation, their mechanical analysis is somewhat unclear and general, which cannot lead to the formation of mathematical modeling thoughts. In fact, Mechanical Representation is a process of integrating the consisting elements of mathematical modeling problems and putting them into certain theoretical frames. It's the key not only to solving problems, but also to distinguishing subjects' different levels of mathematical modeling representation. In the guidance of Mechanical Representation with generality, abstraction and mobilization features, excellent students are able not only to search for and select specific information from problems, but also to identify problem patterns correctly and find rational routes of mathematical modeling. However, normal students find it hard to be guided by Mechanical Representation. As a result, their representation was trivial and disordered, and they did not articulate correct mathematical modeling thoughts. Some research (Xin, 2004; Yu, 2005; Zhong, Chen, & Zhang, 2009) found that solving process and results are influenced by different problem representation modes; the representation modes adopted is a key factor affecting the efficiency of problem solving.

Differences in Adoption of Mathematical Modeling Strategies

This research found that there is obvious difference between excellent and normal students' adoption of Mathematical Modeling Assumption, Mathematical Modeling Construction, Mathematical Modeling Self-monitoring, Mathematical Modeling Validation, and Mathematical Modeling Adjustment Strategy. Previous research (Eysenck & Keane, 2009) indicates that schema richness and levels have all effect on representation quality, and representation quality have all effect on selecting strategies. That is, schema affects the search, selection, production and implementation of strategies through representation. The higher the schema level is, the higher the strategy level is. Having rich cognitive schema, excellent students implement plural representation of mathematical modeling problems. Problem representation systems have been formed which provide a basic condition for activation, search, production and application of mathematical modeling strategies. Excellent students have rich problem schemas, so they have a great deal of problem modes for reference and analogy. Therefore, when constructing mathematical modeling, they first adopt Pattern Recognition Match Strategy. If there is no given model, then they use the Sample Analog Strategy to search for modes similar to or related to current problems in the schema, analyze the problem structure, construct a model and adopt methods and set them for reference, thus gaining the thinking pattern and method of modeling. On the other hand, though adopting Pattern Recognition Match Strategy as well, normal students' problem schemas are relatively poor, it is difficult for them to gain the applicable problem modes, nor can they find sample problems similar to or related to current problems. When failing to adopt Pattern Recognition Match Strategy, they often tend to use instant Generation Strategy. At this stage they always search blindly in the schema and seldom use problem patterns and methods for reference. As a result, it is difficult for them to find a right thinking pattern and method for mathematical modeling. When facing obstacles, excellent subjects usually try to find whether there is any problem in their assumption at first, and then try to change their assumption or make new assumption. They may also maintain the previous assumption, but analyze and adjust the modeling method. However, the normal students tend to validate and adjust the methods of modeling solving, but to ignore their assumption and methods of modeling. Actually, the adoption of the Assumptive Adjustment Strategy and the Modeling Methodical Adjustment Strategy can effectively improve the probability of successful mathematical modeling.

Differences in Thinking Patterns

This research finds that there are significant differences in thinking patterns, results and efficiency of mathematical modeling between excellent students and normal students. These differences may be the results of the subjects' cognitive structures of mathematics and science, representation levels, information processing styles, and levels of mathematical modeling strategy adoption.

In difference in cognitive schema between experts and novices is the basic reason for their difference in problem solving ability (Sweller, Chandler, & Tiemey, 1990). Excellent students' cognitive structure of mathematics and science contains rich cognitive schema, which enable the excellent students to make assumptions beyond the given information. Once the cognitive schema is activated, it can lead the student to search mathematical modeling problems in a certain mode, and look for related features. As a result, the procedures and methods of solving mathematical modeling problems are easily developed. Cognitive schema contained in normal students' cognitive structure is less, and the knowledge in cognitive schemata is seldom organized, with its units coupled with each other loosely and with little declarative and procedural knowledge contained. So they can only develop simple representation on the basis of the superficial similarity of mathematical modeling problems, and can hardly find effective solving procedures and methods.

The levels of subjects' representation determine their selection and application of mathematical modeling strategies, which directly further influences the rationality, effectiveness and correctness of their mathematical modeling thinking patterns, and influences the process and results of mathematical modeling solving as a result. The knowledge in excellent students' schemata is well organized; the units of knowledge, including the declarative knowledge of related fields, are highly related. So in the problem solving process, excellent students can form complicated representation similar to the problem structure, and this representation provides an important basis for the selection and application of appropriate strategies.

There are differences in information processing modes between excellent students and normal students. When dealing with related information, excellent subjects are able to pick up the stored information from the long-term memory system, process it and find a solution. Previous research (Liang, 1997; Sternberg, 1995) indicates that the modes of focusing on and encoding problems are different between experts and novices. Novices perceive problems from the surface structure, while experts solve problems according to rules and principles. From the analysis of protocol materials, we know that the protocol materials of excellent subjects are comparatively simple with several smooth problem-solving steps. It indicates that they can quickly retrieve related information to process from their long-term memory system and find answers fast when dealing with new information. The time excellent students spend in solving problems is much less than that spent by normal students. This phenomenon may have something to do with the “automation” in some procedures in the process of solving mathematical modeling problems. It makes many intermediate steps in the problem-solving process not appear in short-term memory. On the other hand, when solving mathematical modeling problems, normal students always pay conscious attention to the relationship of each condition, especially the intermediate steps, so they prolong the time spent and reduce the efficiency. The reason why excellent students spend more time on complicated problems than do normal students is that some subjects gave up exploring the problem before they were done.

Subjects’ levels of selecting and adopting strategies directly determines the effectiveness of the adopted strategies, which further influences the rationality and effectiveness of mathematical modeling thinking patterns, and finally affects the results and efficiency of mathematical modeling.

Reliability and Validity of the Protocol Materials

In order to increase the objectivity of the protocol materials analysis, we discriminated, classified and analyzed subjects’ protocol materials of the tests and interviews twice (with a three months’ interval between them), mainly in the aspects of problem representation, strategy adoption, thinking patterns, modeling results and problem-solving efficiency. The result shows that the consistency coefficient between the two analyses is 0.902, which indicates that the reliability of the testing materials is high.

To insure that the protocol analysis methods and expert-novice comparison methods can deeply analyze the cognitive features and possible differences of excellent and normal students, the researchers made predictions often when working out the testing materials of mathematical modeling presentation. The testing problems have been adjusted several times, and different levels of difficulty were prepared, aiming at students at different modeling levels providing opportunities for presenting their thinking patterns and possible differences when faced with mathematical modeling problems at different levels of difficulty. The analysis of the testing results does show that characteristics and differences exist in problem representation, strategy adoption, thinking patterns, modeling results and problem-solving efficiency between excellent and normal students. This indicates that the testing materials can effectively measure the features and differences of excellent and normal students’ mathematical modeling, and that the validity of the testing problems is satisfactory.

Conclusion

Conclusion is drawn on the research range and conditions: excellent and normal students represent different cognitive features in aspects of problem representation, strategy adoption, thinking patterns, modeling results and problem-solving efficiency. The conclusion is represented with the following points: (1) regarding modes, span and methods of modeling problem representation: both excellent and normal students have adopted Symbolic Representation and Methodical Representation, but excellent students tend to use more Mechanical Representation; excellent students are inclined to use Plural Representation while normal students are inclined to use Single Representation; excellent students are inclined to use Circular Representation while normal students are inclined to use One-way Representation. (2) regarding strategy adoption of mathematical modeling: excellent students tend to use Equilibrium Assumption Strategy, while normal students tend to use Accurate Assumption Strategy; excellent students tend to use Sample Analog Strategy, while normal students tend to use Instant Generation Strategy; excellent students tend to use Instant Monitoring Strategy, while normal students tend to use Review Monitoring Strategy; excellent students tend to use Theoretical Deduction Strategy and Intuitive Judgment Strategy, while normal students tend to use Data Validation Strategy; excellent students tend to use Assumptive Adjustment Strategy and Modeling Methodical Adjustment Strategy, while normal students tend to use Model Solving Adjustment Strategy.(3) regarding modeling thinking patterns, results and efficiency: excellent students' verbal protocol is relatively simple, their expressions are of strong logic, analyses are profound and clear, thinking patterns are multiple and fast, their adoption of mathematical modeling methods refers to heuristic search, they are more likely to gain correct results and achieve higher efficiency. On the contrary, normal students' verbal protocol is relatively complicated, their expressions lack inner logical connection, analyses are superficial and fuzzy, thinking patterns are slow and inflexible, their adoption of mathematical modeling methods refers to blind search, and they are more likely to achieve wrong results and low efficiency.

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