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"American Pi": The Story of a Song about Pi

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This paper begins by overviewing motivations and means for using music in the teaching of mathematics – in particular, six roles for the use of song. We then share inspirations and variations for the award-winning song "American Pi" (which parodies a song that topped the charts in the United States, Australia, Canada, and New Zealand), followed by overviewing several options for implementation in the mathematics classroom, especially the high school classroom. It is hoped that focusing on characteristics and trajectory of one particular mathematics song may help yield a framework or context for examining, using, and writing other mathematics songs.

Key words: Pi, lyric, song, humanistic mathematics, mathematics history.

Motivations

There are many ways music can be used to motivate or facilitate the learning of mathematics. Robertson and Lesser (2013) include many references of books and articles that facilitate classroom explorations of the mathematical concepts embedded in music composition, instrument construction, or acoustics. Robertson and Lesser also provide detailed descriptions of in-class acoustic guitar explorations (translations, the locations of frets and harmonics, and Mersenne's laws for the frequency of oscillation of a stretched string) that are accessible to high school students, and can also be expanded or simplified for classes that are older or younger, respectively.

Another type of connection between mathematics and music is to write or analyze songs whose lyrics connect with mathematics. While most examples of such songs seem to target elementary school students (e.g., helping children learn multiplication facts), there is also precedent for using mathematics songs with students in subsequent grades. In fact, it could be argued that there is a greater need at those higher grades for an innovative intervention, if that is when students (especially females and minorities) are at greater risk for losing interest or support for reaching their full potential in the subject. Lesser, An, and Tillman (in press) cite examples across many disciplines of how song may increase student learning, recall, and motivation, as well as reduce stress or anxiety.

Lesser (2014) discusses how mathematics songs can have many possible uses, including: (1) aiding recall (of procedures, properties, definitions, digits of pi, etc.), (2) introducing concepts or terms, (3) reinforcing

mathematical thinking processes (e.g., the Pólya (1945) four-step heuristic for problem solving), (4) connecting to history, (5) connecting to the real world, and (6) humanizing mathematics. The author's song "American Pi" (see Appendix 1) is one of those rare songs that arguably accomplishes all six of those uses (as described in the rest of this section) and therefore is an appropriate choice for illustrating and exploring the potential of song in mathematics education.

The chorus of "American Pi" contains a mnemonic for the first six significant figures of π (enough for almost any practical purpose), thus accomplishing a recall objective. Also, several terms are introduced by the lyric such as rational, transcendental, and continued fractions. Archimedes' method (called the "classical method of computing pi" by Eves, 1990) of bounding circumference by perimeters of inscribed and circumscribed polygons with increasing numbers of sides is an example of a mathematical thinking process (see item #7 of Appendix 2).

The song makes tangible connections to history by noting values of π derived from the Bible (e.g., I Kings 7:23-26; II Chronicles 4:2), Archimedes' method, arctangent series, continued fractions (in the 1600s), the decision by the English in the 18th century to denote the circle ratio by the Greek letter π, Lambert proving (in 1767) that π was irrational, and Lindemann proving (in 1882) that π was transcendental, which showed the impossibility of "squaring the circle" (i.e., given a circle, finding the side of a square so that the square and circle have equal areas; this requires constructing $\sqrt{\pi}$ using only compass and straightedge). United States students who think of π's history as being fully settled and known may be surprised to learn that there is ongoing discussion (e.g., Richeson, 2015) about who (probably Archimedes, but we do not really know) first recognized that circumference divided by diameter is a constant. Those students who associate π only with Greece or other non-US countries in the distant past are surprised to learn about the 1897 Indiana state legislature's consideration of House Bill no. 246 that would have effectively legislated a value of either 3.2 or 4 for π (Hallerberg, 1977). That latter story about the foibles of public figures considering instituting an incorrect value of π also goes a long way toward humanizing our subject.

Inspirations

My inspiration to write "American Pi" came in 1997 while teaching (at the University of Northern Colorado) mathematics history, a course I had not taken as a student. In the course textbook (Eves, 1990), I was fascinated by the half-dozen pages of annotated timeline about π. Its rich diversity of people, places, and procedures made me think about songs that are packed with rapid-fire allusions to events, such as Billy Joel's 1989 #1 hit "We Didn't Start the Fire" and Don McLean's 1971 song "American Pie". "American Pie" was released in a rhythmic, abbreviated form in 2000 by Madonna and was a #1 hit

in several countries (as McLean's version was), thus giving the song a new boost of recognition among younger listeners. A lesson plan in the Rock and Roll Hall of Fame and Museum collection asks high school students to identify as many historical allusions in "American Pie" as they can, such as the 1959 death of Buddy Holly, customs/values of the 1950s, John Lennon, Rolling Stones, Bob Dylan (and his motorcycle accident), rock and roll, etc.

While most of my non-mathematics songs have original music and lyrics, most of my mathematics songs are parodies of existing songs because they are quicker to write and easier for listeners to follow (since familiar melody and structure are already in place), and have the dimension of additional humor based on what changes to the lyric are made (or not made). I chose the Don McLean song as the vehicle for several reasons, including: (1) the pi homonym in the title, (2) the distinctively American angle on the history of π, (3) the lyrical structure and length of the song which lends itself to a stream of historical references, (4) the song is uptempo enough to be lively and engaging, but not so rapid-fire as to be difficult to sing, and (5) the digits of π were an excellent fit with the meter and rhyme in the chorus, which was the initial seed for the song. From the additional parodies of "American Pie" that a Google search yields, it appears that some of these criteria have been noticed by others as well.

We unpack (2) by noting that an American contribution to the history of π is reflected in the second verse's referencing the previously-mentioned American footnote in π's history with the 1897 Indiana legislature (Hallerberg, 1977). Also, Pi Day (3/14) is very much an "American holiday" in the sense that the United States is virtually the only country which writes the calendar date as month/day/year instead of day/month/year. Launched at San Francisco's Exploratorium (thanks to physicist Larry Shaw) in 1988, Pi Day was officially recognized in 2009 when the US House of Representatives passed House Resolution 224 which "encourages schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics." (http://www.gpo.gov/fdsys/pkg/BILLS-111hres224eh/pdf/BILLS-111hres224eh.pdf)

Variations

Of the 60+ mathematics lyrics I have published in periodicals (e.g., http://www.math.utep.edu/Faculty/lesser/GreatestLESSERhits.html or Lesser (2014)), "American Pi" has received the most requests for (re)publication, including: *Mathematics Teacher*, *Pi in the Sky*, *Journal of Irreproducible Results*, *The Pi*, *Convergence*, Chase (2001), and Posamentier and Lehmann (2004). A significantly revised version was published in *Math Horizons* thanks to suggestions from then-editor Art Benjamin, who in turn has used with permission a version of the song in a 24-lecture video course (starting at the 29:35 mark of Lecture 12 in course 1411, *The Joy of Mathematics*, The

Teaching Company, 2007), a book (*The Magic of Math: Solving for x and Figuring Out Why*, Basic Books, 2015), and his live mathemagician performances. Benjamin playfully works many more digits into a final chorus by rapidly singing (beats in boldface) "3 point **1** 4 1 5 **9** 2 6 5 **3** 5 8 **9**".

While earlier versions of the song varied the chorus each time to tuck in yet more π facts and history, I gravitated to a version that keeps the chorus the same each time. This decision for aesthetic simplicity makes it easier for people to recall and sing the words (which, in turn, better facilitates community and learning).

Sometimes other people's uses of "American Pi" are quite novel. Calvin Coolidge (a band of Clevelanders in high school at the time) sings the pre-verse of Don McLean's original "American Pie" before launching into my "American Pi": http://www.youtube.com/watch?v=Ll_45NomcFk. North America's only mathematics museum, the National Museum of Mathematics (momath.org; see Henebry, 2012), sponsored a π song/poem contest in honor of March 14, 2015 being "Pi Day of the Century" (since 3/14/15 is this century's only date containing the first five digits of π). Almost 200 entries were received and "American Pi" was announced (at 3:14pm on Pi Day at the Museum) as winning a first-place award in the adult division and this news attracted much media coverage in the author's city (e.g., Candelaria, 2015; Martinez, 2015; Masterson, 2015). Interestingly, Don McLean's public debut of "American Pie" happened on a March 14 (Wikipedia, 2015). "American Pi" can be heard at http://www.math.utep.edu/Faculty/lesser/AmericanPi.mp3 or at http://momath.org/home/americanpi/. Appendix 1 has the version of the lyric that won, with an optional last verse appended, and with a slight refinement of the line about Lambert inspired by feedback from an anonymous reviewer for this journal.

Other songs about π include "Circle Song" and "Pi Will Go On" (see Lesser, 2014), the latter inspired by π having a decimal representation whose digits never end or repeat by virtue of π being an irrational number, and the former inspired by the desire to help students distinguish and recall the two most common formulas associated with circles. Also, the math club of an independent K-12 day school in a Cleveland suburb turned the very digits of π into melody (e.g., Lewellen, 1987) by assigning the digits 0 through 9 to the notes B3, C4 (middle C), D4, E4, F4, G4, A4, B4, C5, and D5, respectively. Interestingly, on March 14, 2012, a U.S. District Court judge dismissed a copyright infringement suit relating to a pi-based melody by ruling that "pi is a non-copyrightable fact and the transcription of pi to music is a non-copyrightable idea" (Ornes, 2012).

Implementations

There are many ways a high school or college teacher can use a song such as "American Pi" with students, either as part of a Pi Day celebration or

as part of a unit for a geometry or mathematics history lesson. Even the playing of the song has several options, ranging from pressing PLAY from an online soundfile (referenced previously) to a live performance using the teacher or student talents (or karaoke) in the room. Students can be further engaged by being invited to sing along or by doing some research in advance to be able to fill in some blanks that could be deliberately left in the lyric, as illustrated with the verse below. (The author's current National Science Foundation grant Project SMILES, NSF/EHR/DUE 1544237, is developing interactive fill-in-the-blank songs to be assessed for effectiveness in helping students learn introductory statistics.)

> In the Bible we do see the circle ratio appears as ___,
> Or a little more....
> That genius _____ found with polygons, an upper bound
> Of ____ for sure!
> The Chinese got it really keen: 3 5 5 over ___!
> More joined the action with continued _____.
> In the 1700s, my oh my, the English coined the symbol ___,
> Then _____ showed it was a lie to look for rational pi.

A high school mathematics teacher in Illinois reports that "In preparation of Pi Day,…I share the lyrics of your 'American Pi' with my Algebra 1 students…. This class then spends part of a week researching the topics you address in your lyrics. The students then turn their results into either a PowerPoint, book, or short story….The really neat part about this project is that in researching the lyrics, they end up finding all kinds of other facts about pi along the way." (Satterlee, 2008).

Another possibility is to have students make their own creations, which many students did for their age group category in the Museum of Mathematics Pi Day contest (http://momath.org/home/2015-pi-day-winners/). Students might write a new "American Pi" verse that connects to some other aspect of π they are learning about in school (e.g., how π shows up in formulas for areas and volumes of cylinders, spheres, and cones) or something that they research for enrichment (e.g., the Buffon needle problem). For classes so inclined, a few phrases of the song may even lend themselves to kinesthetic movements such as having "circle ratio appears as 3" accompanied by sweeping one's hand in a circle, tracing its horizontal diameter, and then holding up three fingers.

Another way the song can be used is to assign students problems that connect to the song they have just encountered. Appendix 2 offers a list of possible questions from which teachers can choose, all of which require at least middle school mathematics and most require only high school geometry and algebra. Indeed, question #7 is very similar to a "Develop π" lab (in the high school geometry textbook Burger et al., 2007, p. 599) which asks students to construct a similar figure and use the squares' perimeters to yield bounds for π.

A subsequent exercise on that page uses perimeters of inscribed and circumscribed hexagons, just like Archimedes! That lab is designed to be used with the textbook section on developing area and circumference formulas for circles and an area formula for regular polygons. Other Appendix 2 items offer a vehicle for enrichment. For example, the topic of continued fractions is not generally covered in a high school or college mathematics course and yet it is a topic that is quite interesting and many aspects of it are quite accessible to students.

The use of the song should depend upon the learning goal of a given lesson. For example, if one is not concerned with making historical connections and wants merely a good mnemonic for the first six significant figures of π, then it could suffice to just take 18 seconds to sing one chorus from "American Pi". Another song that could serve this mnemonic-only purpose is Kevin Ferland and Bill Calhoun's "3.14159/Circle" (parodying the Tommy Tutone 80s hit "867-5309/Jenny"), whose chorus ends "three point one four one five ni-ee-yine". An example yielding four additional significant figures excerpts the tune of Jimmie Dodd's "Mickey Mouse Club Theme Song" (where it spells out "M-I-C, K-E-Y, M-O-U-S-E"): "3 point 1, 4-1-5, 9-2-6-5-3". Any of these musical choices may be a more engaging mnemonic than, say, a sentence in which digits of π are conveyed by the number of letters in each word of the sentence.

Conclusions

Lesser (2014) and Lesser, An, and Tillman (in press) review and report promising examples of using song effectively in educational settings. Also, my song "Hotel (Called) Infinity" (e.g., Glaz, 2011) was used in professional development work to summarize and explore mathematical ideas of the infinite (Sponsel, 2010). It is also not hard to conceptualize a use for song in a research study given that the Hilbert's Infinite Hotel metaphor used in my song was also used in research (Mamolo & Zazkis, 2008) to assess students' understanding of infinity. Beyond individual studies is the broader potential benefit of how math-positive songs may help change a culture still filled with shirts (McCaughey, 2011) and songs (e.g., Jimmy Buffett's "Math Suks"; see Oldenburg, 1999) that reflect negative images of our discipline.

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Appendix 1: "American Pi" lyric

Prologue: A long, long time ago I can still remember
How that math sure used to make me smile.
And I knew if I had my chance, I would ace geometry class
And make my parents happy for a while.
But some math books made me shiver--
Dry procedures, all delivered:
Nothing past the rational,
And nothing transcendental.
I can't remember if I cried, reading 3.14159…
But something touched me deep inside
The day I learned of π.… so:

CHORUS: Find, find the value of π, starts 3.14159
A good ol' fraction you may hope to define,
But the decimal never dies, never repeats or dies…

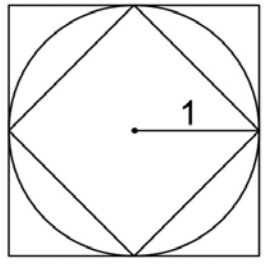
In the Bible we do see the circle ratio appears as 3,
Or a little more....
That genius Archimedes found with polygons, an upper bound
Of 22/7 for sure!
The Chinese got it really keen: three-five-five over one-thirteen!
More joined the action with continued fractions.
In the 1700s, my oh my, the English coined the symbol π,
Then Lambert proved that you and I can't find rational π.
He started singing.... (*Repeat Chorus*)

Late 1800s, Lindemann shared why a circle can't be squared
But some folks tried anyway--
Like the Indiana doctor who said π was 4 or 3.2
And thought his proof should be a law someday.
The Indiana congressmen read his paper there and then
A bill got through the House by a vote unanimous!
But in the end, the statesmen cried, "It's not for us to decide,"
So the bill was left to die like the quest for rational π.
Let's try singing.... (*Repeat Chorus*)

That doctor's pi-in-the-sky dreams may not look so extreme
'Cause we long believed
Deductive systems could be complete and there was one true geometry.
But now there's more we see
Now when it comes to π, we test the best machines to find
Many trillion places that so far lack pattern's traces.
It's great when we can truly see math as human history--
That adds curiosity: easy as π!
Let's all try singing.... (*Repeat Chorus*)

Appendix 2: class questions, in order of appearance in "American Pi"

1. What makes a number irrational and can you name one besides pi?
2. What makes a number transcendental? Can you name one besides pi?
3. What is the difference between a decimal and a fraction and what are the advantages of each?
4. What is an example of a fraction where the digits of its decimal representation terminate?
5. What is an example of a fraction where the digits of its decimal representation go into a repeating pattern?
6. How does 1 Kings 7:23 suggest a value of 3 for pi? (extension: see Lesser (2006) to learn how further information can yield a closer value for pi)
7. Recreate how Archimedes started his process by using appropriate tools to construct a circle of unit diameter (so the circumference will equal π), and then find the perimeters of the inscribed and circumscribed hexagons. Those perimeters are lower and upper bounds for π.



To explore a similar, but simpler, version of Archimedes' process, explore this diagram and use the perimeters of the squares and the unit circle to verify that because the circumference of a unit circle is 2π, then 2π is bounded by $4\sqrt{2}$ and 8, which places the value of π between 2.8 and 4.

8. Verify how many decimal places of accuracy the Chinese approximation 355/113 provides.
9. Look up what a "continued fraction" is. Find the simplified improper fraction represented by this truncated continued fraction: $3 + 1/ (7 + (1/ (1/1))$
10. Look up why the Greek letter π was an appropriate choice (by English mathematics teacher William Jones in 1706) of a symbol for the circle ratio.
11. With respect to the 1761 Johann Lambert proof that pi is irrational, consider the result that "if x is rational, then $\tan(x)$ is irrational". That is logically equivalent to the contrapositive statement "if $\tan(x)$ is rational, then x is irrational". Now let $x = \pi /4$ and see if you can deduce that π is irrational.
12. Squaring the circle means constructing a square with the same areas as a given circle using a finite number of steps with the compass and straightedge. Given a circle with unit radius, what is the circle's area? For a square to have that same area, what would the length of a side of the square have to be? Can the answer to that last question be the root of a polynomial with rational number coefficients?
13. Use Hallerberg (1977) to explain how 4 and 3.2 were obtained.
14. What is a deductive system and what did Kurt Gödel prove in 1931 about completeness?
15. For about 2000 years, Euclidean geometry was assumed to be the only "true" geometry. What is another geometry we now know to be just as valid

and logically consistent? How does that geometry differ from Euclidean geometry in terms of the nature of parallel lines?

16. Look up how many digits of π are currently known.

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